A Working Vacation Queue in Moscow that is Partially Broken Down and Whose Arrival Rate Depends on the Server State

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Abstract: Consideration has been given to a single server Markovian queueing system that alternates between operating in regular busy state, repair state, and working vacation state. The system operates as a single server Markovian queue when it is busy. It operates once again as a single server Markovian queue with varying arrival and service rates when it is undergoing maintenance or vacation. The vacation time is negative exponential, and the vacation policy is multiple vacation policy. Furthermore, if a server malfunctions while being used, repairs are initiated right away. Two distinct negative exponential distributions are used to represent the breakdown and repair periods. The server provides the customer with service during the repair period at a rate that is negatively exponentially different from the service rate. Additionally, several interesting cases are given.

Keywords: Working vacation, arrival rate based on state, and Matrix-Geometric technique.

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1. Introduction

Queuing and breakdown are two characteristics shared by the computer system, industrial system, production system, and communications network. In actuality, a queueing system with breakdown may be used to create these kinds of systems with these two characteristics. Numerous investigators have examined these models, such as Federgruen and Green (1986), Li et al. (1997), Tang (1997), Nakdimon and Yechiali (2003), Wang et al. (2007), Wang et al. (2008), and Choudhry and Tadj (2009)

In day to day life, it can be seen that the server works during his vacation period, if the necessity occur, called working vacation queue. In the working vacation queues, the server works with variable service rate, in particular reduced service rate, rather than completely stops service during vacation period. Servi and Finn(2002) have first analyzed an$M/M/1$ queue with multiple working vacation, in which the vacation times are exponentially distributed. Wu and Takagi(2006) extend the work to an $M/G/1$ queue. Kim et al.(2003) analyzed the queue length distribution of the $M/G/1$ queue with working vacations. Liu et al.(2007), examined stochastic decomposition structure of the queue length and waiting time in an $M/M/1$ working vacation queue. Xu et al.(2009) extended the $M/M/1$ working vacation queue to an $M^X/M/1$ working vacation queue. Under certain assumptions, Li et al. (2009) analyzed an M/G/1 queue with exponential working vacation using the matrix analytic approach. A multi-server queue with a single working vacation is considered by Lin and Ke (2009). An individual working vacation model with server failure was studied by Jain and Jain (2010). In recent years, Ke et al. (2010) have provided a brief overview of vacation models. Yechiali and Naor (1971) considered a single-server exponential queuing model with arrival state depending on operational state or breakdown state of the server. This is because it is common to observe that both customer arrival and service depend on the current state of the system, i.e., number of customers in the system, etc.Fond and Ross (1977) calculated the steady-state percentage of lost customers by analyzing the same model on the premise that every arrival who discovers the server is busy is lost. A single server queueing model with arrival rate dependent on server status has been studied by Shogan (1979). Shanthikumar (1982) examined a Poisson queue with a single server and an arrival rate that varied based on the server's condition. A broad bulk service queue whose arrival rate was contingent on server failures was examined by Jayaraman et al. in 1994. The topic of queueing systems with variable arrival rates was covered by Tian and Yue (2002). The authores studied the model by using the principle of quasi-birth and death process(QBD) and matrix-geometric method. Furthermore, they calculated some performance measures, such as the number of customers in the system in steady-state, etc., The Matrix-geometric technique approach is a helpful resource for handling more difficult queueing issues. Numerous scholars have used the matrix-geometric technique to tackle a variety of queueing issues in various contexts. Different matrix geometric solutions of stochastic models were explained by Neuts (1981). The computable explicit formula for the probability distributions of the queue length and other system characteristics is developed using a matrix-geometric technique.

In this research, we take a fresh approach by assuming that the server continues to function during the breakdown phase, although more slowly. State-specific factors also affect the arrival rates. In addition, the server goes on vacation anytime the system is empty, and it stays on vacation until at least one customer is waiting to be served. If any clients arrive during the vacation time, the server will serve them more slowly.

In this paper, we consider an $M/M/1$queuewith multiple working vacation and with partial breakdown. The arrival rate dependents on the server states. The matrix geometric approach has been used to study the model. The remainder of this essay is structured as follows: We describe the model and establish its quasi-birth-death process in section 2. We use the matrix geometric technique to describe the steady state solution in section 3. We provide some system performance metrics in section 4. Section 5 provides a few specific models. and In section 6, we carried out a numerical study.

2. The Model

We consider a single-server queueing system with the following characteristics:

1. The system alternate between two states, up state and down state. In the up state it is either in regular state or in working vacation state. In the down state it is in the repair state.
2. Arrival process follows Poisson.
3. When the system is in regular busy period it serves customers based on exponential distribution with rate μ.
4. During the regular busy period the arrival parameter is λ.
5. The server takes vacation, if there are no customer in the queue at a service completion point.
6. During vacation, the arrival rate is$\lambda_1 (\lambda_1 < \lambda)$.
7. Vacation period follows negative exponential distribution with rate θ and the vacation policy is multiple vacation policy.
8. When the server is in vacation, if customer arrives, the server serves the customer using exponential distribution with rate$\mu_1 (\mu_1 < \mu)$. The server may break down during a service and the break downs are assumed to occur according to a Poisson process with rate α.
9. Once the system break downs, the customer whose service is interrupted goes to the head of the queue and the repair to server starts immediately.
10. Duration of repaired period follows negative exponential with rate β.
11. During repair period customers arrive according to Poisson process with rate λ_2 $(\lambda_2 < \lambda_1)$.
12. During repair period the server serves the customers, and the service period follows negative exponential with rateμ_2 $(\mu_2 < \mu_1 < \mu)$.
13. The first come first served (FCFS) service rule is followed to select the customer for service.

2.1 The quasi-birth-and-death (QBD) process:

The model defined in this article can be studied as a QBD process. The following notations are necessary for the analysis:

Let $L(t)$ be the number of customers in the queue at time t and let

$$J(t) = \begin{cases} 0, if the server is on working vacation \\ 1, if the server is busy \\ 2, if \text{ the server is on partial breakdown} \end{cases}$$

be the server state at time t.

Let $X(t) = (L(t), J(t))$, then $\{(X(t)): t \geq 0\}$ is a Continuous time Markov chain (CTMC) with state space $S = \{(i,j): i \geq 0; i = 0,1,2\}$, where i denotes the number of customer in the queue and j denotes the server state.

Using lexicographical sequence for the states, the rate matrix$Q$, is the infinitesimal generator of the Markov chain and is given by

$$Q = \begin{bmatrix} B_0 A_0 & & & \\ & A_2 A_1 A_0 & & \\ & A_2 A_1 A_0 & & \\ & A_2 A_1 A_0 & & \\ & & \cdot\ \cdot\ \cdot & \\ & & \cdot\ \cdot\ \cdot & \\ & & & \cdot\ \cdot\ \cdot \end{bmatrix}$$

where the sub-matrices A_0, A_1 and A_2 are of order 3×3 and are appearing as

$$A_0 = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda_2 \end{bmatrix}$$

$$A_1 = \begin{bmatrix} -(\lambda_1 + \mu_1 + \theta) & \theta & 0 \\ 0 & -(\lambda + \mu + \alpha) & \alpha \\ 0 & \beta & -(\lambda_2 + \mu_2 + \beta) \end{bmatrix}$$

$$A_2 = \begin{bmatrix} \mu_1 & 0 & 0 \\ 0 & \mu & 0 \\ 0 & 0 & \mu_2 \end{bmatrix}$$

and the boundary matrix is defined by

$$B_0 = \begin{bmatrix} -(\lambda_1 + \theta) & \theta & 0 \\ \mu & -(\lambda + \mu + \alpha) & \alpha \\ 0 & \beta & -(\lambda_2 + \beta) \end{bmatrix}$$

We define the matrix$A$ as$A = A_0 + A_1 + A_2$. This matrix A is a 3×3 matrix and it is of the form

$$A = \begin{bmatrix} -\theta & \theta & 0 \\ 0 & -\alpha & \alpha \\ 0 & \beta & -\beta \end{bmatrix}$$

3. The Steady State Solution

Let $P = (p_0, p_1, p_2, \dots)$ be the stationary probability vector associated with Q, such that $PQ = 0$ and $Pe = 1$, where e is a column vector of 1's of appropriate dimension.

Let $p_0 = (p_{00}, p_{01}, p_{02})$ and $p_i = (p_{i0}, p_{i1}, p_{i2})$ for$i \geq 1$.

If the steady state condition is satisfied, then the sub vectors p_i are given by the following equations:

$$p_0 B_0 + p_1 A_2 = 0 \qquad (1)$$

$$p_i A_0 + p_{i+1} A_1 + p_{i+2} A_2 = 0, i \geq 0 \qquad (2)$$

$$p_i = p_0 R^i; i \geq 1 \qquad (3)$$

where R is the rate matrix, is the minimal non-negative solution of the matrix quadratic equation (see Neuts(1981)).

$$R^2 A_2 + R A_1 + A_0 = 0, \qquad (4)$$

Substituting the equation (3) in (1), we get

$$p_0 (B_0 + R A_2) = 0 \qquad (5)$$

and the normalizing condition is

$$p_0 (I - R)^{-1} e = 1 \qquad (6)$$

Theorem: 3.1

The queueing system described in section 2 is stable if and only if$\rho < 1$, where

$$\rho = \frac{\lambda_2 \alpha + \lambda \beta}{\mu \beta + \mu_2 \alpha}$$

Theorem: 3.2

If$\rho < 1$, the matrix equation (4) has the minimal non-negative solution

$$R = -A_0 A_1^{-1} - R^2 A_2 A_1^{-1}$$

Proof

Since the matrix A is reducible. The analysis present in Neuts(1978) is not applicable. In Lucantoni(1979), similar reducible matrix is treated for the case when the elements are probabilities.

Equation (4) can be written as

$$A_0 A_1^{-1} + R A_1 A_1^{-1} + R^2 A_2 A_1^{-1} + 0. A_1^{-1}$$

Since $A_1$is non-singular, A_1^{-1} exists and

$$R = -A_0 A_1^{-1} - R^2 A_2 A_1^{-1} \qquad (7)$$

where,

$$A_1^{-1} = \begin{bmatrix} \frac{-1}{(\lambda_1 + \mu_1 + \theta)} & \frac{S_0(\lambda_2 + \mu_2 + \beta)\theta}{(\lambda_1 + \mu_1 + \theta)} & \frac{S_0 \alpha \theta}{(\lambda_1 + \mu_1 + \theta)} \\ 0 & S_0(\lambda_2 + \mu_2 + \beta) & S_0 \alpha \\ 0 & S_0 \beta & S_0(\lambda + \mu + \alpha) \end{bmatrix}$$

$$S_0 = \frac{1}{[\alpha\beta - (\lambda + \mu + \alpha)(\lambda_1 + \mu_1 + \beta)]}$$

Using Nuets and Lucantoni(1979), the matrix R is numerically computed by using the recurrence relation with $R(0) = 0$ in equation (7).

Theorem: 3.3

If $\rho < 1$, the stationary probability vectors $p_0 = (p_{00}, p_{01} p_{02})$ and$p_i = (p_{i0}, p_{i1} p_{i2}); i \geq 1$ are

$$p_{00} = \frac{1}{S_1 + S_2[(\lambda_1 + \theta) - \mu_1 r_0] - S_3[(\mu r_{01} + \theta) + \frac{1}{\mu}[(\lambda_1 + \theta) - \mu_1 r_0][\mu}$$

$$p_{01} = \frac{1}{\mu}[(\lambda_1 + \theta) - \mu_1 r_0] p_{00}$$

$$p_{02} = \frac{-1}{(\beta + \mu r_{21})}[(\mu r_{01} + \theta) + \frac{1}{\mu}[(\lambda_1 + \theta) - \mu_1 r_0][\mu r_1 - (\lambda + \mu + \alpha)]] p_{00}$$

and$p_i = p_0 R^i; i \geq 1$

where,

$$S_1 = \frac{1}{1 - r_0} + \frac{r_{21} r_{02} + r_{01}(1 - r_2) + r_{01} r_{12} + r_{02}(1 - r_1)}{(1 - r_0)[(1 - r_1)(1 - r_2) - r_{21} r_{12}])}$$

$$S_2 = \frac{1 - r_2 + r_{12}}{\mu[(1 - r_1)(1 - r_2) - r_{21} r_{12}]}$$

$$S_3 = \frac{1 - r_1 + r_{21}}{(\beta + \mu r_{21})[(1 - r_1)(1 - r_2) - r_{21} r_{12}]}$$

Proof

$p_{00}, p_{01}$and p_{02}follows from the equations (5) and (6).

Remark: 3.1

Even though R in Theorem 3.2 has a nice structure which enables us to make use of the properties like$R^n = \begin{bmatrix} r_0^n & r_{01} \sum_{j=0}^{n-1} r_0^j r_1^{n-j-1} \\ 0 & r_1^n \end{bmatrix}$, for $n \geq 1$, due to the form of $r_0 \& r_{01}$, it may not be easy to cary out the computation required to calculate the p_i and the performance measures.

$$R(0) = 0 \qquad (8)$$

$$R(n+1) = -A_0 A_1^{-1} - [R(n)]^2 A_2 A_1^{-1} \text{ for n} \geq 0 \quad (9)$$

and is the limit of the monotonically increasing sequence of matrices $\{R(n), n \geq 0\}$.

4. Performance Measures

(i) Mean queue length $E(L) = p_0 R (I - R)^{-2} e$

(ii) $E(L^2) = p_0 R (I + R)(I - R)^{-3} e$

(iii) Variance of queue length$L = var(L)$

$= [p_0 R(I+R)(I-R)^{-3}e] - [p_0 R(I-R)^{-2}e]^2$

(iv) Probability that the server is ideal $= p_0 e$

(v) Mean queue length when the server is an vacation period $= \sum_{i=0}^{\infty} i p_{i0}$

(vi) Mean queue length when the server is in regular busy period $= \sum_{i=0}^{\infty} i p_{i1}$

(vii) Probability that the server is in working vacation period = pr{J=0}=$\sum_{i=1}^{\infty} p_{i0}$

(viii) Probability that the server is in regular busy period= pr{J=1}=$\sum_{i=1}^{\infty} p_{i1}$

5. Particular Model

In the above model, we assume that $\lambda_1 = \lambda_2 = \lambda$, and$\mu_1 = \mu_2 = \mu$, then we get

$$R = -A_0 A_1^{-1} - R^2 A_2 A_1^{-1}$$

$$p_{00} = \frac{1}{S_1 + S_2[(\lambda+\theta) - \mu r_0] - S_3[(\mu r_{01} + \theta) + \frac{1}{\mu}[(\lambda+\theta) - \mu r_0][\mu r_1 - (\lambda+\mu+\alpha)]]}$$

$$p_{01} = \frac{1}{\mu}[(\lambda+\theta) - \mu r_0] p_{00}$$

$$p_{02} = \frac{-1}{(\beta + \mu r_{21})}[(\mu r_{01} + \theta) + \frac{1}{\mu}[(\lambda+\theta) - \mu r_0][\mu r_1 - (\lambda + \mu + \alpha)]] p_{00}$$

and$p_i = p_0 R^i; i \geq 1$

where

$$A_0 = \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}$$

$$A_2 = \begin{bmatrix} \mu & 0 & 0 \\ 0 & \mu & 0 \\ 0 & 0 & \mu \end{bmatrix}$$

$$A_1^{-1} = \begin{bmatrix} \frac{-1}{(\lambda+\mu+\theta)} & \frac{S_4(\lambda+\mu+\beta)\theta}{(\lambda+\mu+\theta)} & \frac{S_4 \alpha\theta}{(\lambda+\mu+\theta)} \\ 0 & S_4(\lambda+\mu+\beta) & S_4\alpha \\ 0 & S_4\beta & S_4(\lambda+\mu+\alpha) \end{bmatrix}$$

$$S_4 = \frac{1}{[\alpha\beta - (\lambda+\mu+\alpha)(\lambda+\mu+\beta)]}$$

6. Numerical Study

In this section, some examples are given to show the effect of the parameters $\lambda, \lambda_1, \lambda_2, \mu, \mu_1, \mu_2, \theta, \alpha$ and β on the performance measures mean queue length, $E(L^2)$, variance of queue length$L$, For the model examined in this paper, the following variables are relevant: probability that the server is idle, mean queue length during the server's vacation period, mean queue length during the regular busy period, probability that the server is working during its vacation period, and probability that the server is in regular busy period. Case(1), Case(2), and Case (3) represent the equivalent findings.

Case(i): If $\lambda = 0.7, \lambda_1 = 0.5, \lambda_2 = 0.3$, $\mu = 4, \mu_1 = 2, \mu_2 = 1, \theta = 0.6, \alpha = 0.3$ and $\beta = 0.5$, the matrix R is obtained using the equations (8)& (9)

$$R = \begin{bmatrix} 0.182864 & 0.031707 & 0.007446 \\ 0 & 0.166345 & 0.034620 \\ 0 & 0.026950 & 0.192199 \end{bmatrix}$$

and the invariant probability vector is $P = (p_0, p_1, p_2, \dots)$, where

$$p_0 = (0.530459, 0.134089, 0.082768)$$

and the remaining vectors p_i's are evaluated using the relation$p_i = p_0 R^i; i \geq 1$

$p_1 = (0.097001850605, 0.041354890913, 0.024499885738)$

$p_2 = (0.017738146707, 0.010615088977, 0.006862835493)$

$p_3 = (0.003243668471, 0.002513143932, 0.001818602788)$

$p_4 = (0.000593150151, 0.000569907250, 0.000460691052)$

$p_5 = (0.000108465807, 0.000126023864, 0.000112691145)$

$p_6 = (0.000019834491, 0.000027439592, 0.000026829708)$

$p_7 = (0.000003627014, 0.000005916392, 0.000006254289)$

$p_8 = (0.000000663250, 0.000001267717, 0.000001433900)$

$p_9 = (0.000000121285, 0.000000270552, 0.000000324421)$

$p_{10} = (0.000000022179, 0.000000057594, 0.000000072623)$

For the chosen parameters $p_{10} \to 0$, and the sum of the steady state probabilities is found to be 0.955029

The performance measures are

(i) Mean queue length E(L) =0.24836

(ii) $E(L^2)$ =0.410295

(iii) Variance of queue length$L = var(L)$ =0.348612

(iv) Probability that the server is ideal =0.747316

(v) Mean queue length when the server is an vacation period =0.010906

(vi) Mean queue length when the server is regular busy period =0.037819

(vii) Probability that the server is in working vacation period $= pr\{J = 0\}$ =0.010221

(viii) Probability that the server is in regular busy period= $pr\{J = 1\}$ =0.031502

Case(ii): If $\lambda = 0.6, \lambda_1 = 0.4, \lambda_2 = 0.2, \mu = 5, \mu_1 = 3, \mu_2 = 1, \theta = 0.7, \alpha = 0.2$ and $\beta = 0.6$, the matrix R is obtained using the equations (8)& (9)

$$R = \begin{bmatrix} 0.105742 & 0.016102 & 0.002201 \\ 0 & 0.116943 & 0.014977 \\ 0 & 0.013937 & 0.121081 \end{bmatrix}$$

and the invariant probability vector is $P = (p_0, p_1, p_2, \ldots)$, where

$$p_0 = (0.677333, 0.121695, 0.021559)$$

and the remaining vectors p_i 's are evaluated using the relation $p_i = p_0 R^i; i \geq 1$

$$p_1 = (0.082196749747, 0.027048461139, 0.006143921055)$$

$$p_2 = (0.008691648953, 0.004572287668, 0.001329931896)$$

$$p_3 = (0.000919072365, 0.000693185197, 0.000248638971)$$

$$p_4 = (0.000097184551, 0.000099327342, 0.000042510168)$$

$$p_5 = (0.000010276489, 0.000013772967, 0.000006848702)$$

$$p_6 = (0.000001086657, 0.000001871574, 0.000001058144)$$

$$p_7 = (0.000000114905, 0.000000251112, 0.000000158543)$$

$$p_8 = (0.000000012150, 0.000000033426, 0.000000023210)$$

$$p_9 = (0.000000001285, 0.000000004428, 0.000000003338)$$

$$p_{10} = (0.000000000136, 0.000000000585, 0.000000000473)$$

For the chosen parameters $p_{10} \to 0$, and the sum of the steady state probabilities is found to be 0.95269546

The performance measures are

(i) Mean queue length $E(L)$ =0.135078
(ii) $E(L^2)$ =0.174399
(iii) Variance of queue length $L = var(L)$ =0.156153
(iv) Probability that the server is ideal =0.820587
(v) Mean queue length when the server is an vacation period =0.011128
(vi) Mean queue length when the server is regular busy period =0.080998
(vii) Probability that the server is in working vacation period $= pr\{J = 0\}$ =0.010102
(viii) Probability that the server is in regular busy period= $pr\{J = 1\}$ =0.054003

Case(iii): If $\lambda_1 = \lambda_2 = \lambda = 0.6$, $\mu_1 = \mu_2 = \mu = 4, \theta = 0.7, \alpha = 0.2$ and $\beta = 0.6$, the matrix R is obtained using the equations (8)& (9)

$$R = \begin{bmatrix} 0.125 & 0.023709 & 0.001291 \\ 0 & 0.143042 & 0.006959 \\ 0 & 0.020876 & 0.129125 \end{bmatrix}$$

and the invariant probability vector is $P = (p_0, p_1, p_2, \ldots)$, where

$$p_0 = (0.625033, 0.151007, 0.056039)$$

and the remaining vectors p_i 's are evaluated using the relation $p_i = p_0 R^i; i \geq 1$

$$p_1 = (0.078129127622, 0.037589121610, 0.009093811736)$$

$$p_2 = (0.009766140953, 0.007419028785, 0.001536685857)$$

$$p_3 = (0.001220767619, 0.001324858051, 0.000262661662)$$

$$p_4 = (0.000152595952, 0.000223936848, 0.000044711884)$$

$$p_5 = (0.000019074494, 0.000036583675, 0.000007528800)$$

$$p_6 = (0.000002384312, 0.000005842410, 0.000001251367)$$

$$p_7 = (0.000000298039, 0.000000918363, 0.000000205318)$$

$$p_8 = (0.000000037255, 0.000000142717, 0.000000033287)$$

$$p_9 = (0.000000004657, 0.000000021993, 0.000000005339)$$

$$p_{10} = (0.000000000582, 0.000000003368, 0.000000000849)$$

For the chosen parameters $p_{10} \to 0$, and the sum of the steady state probabilities is found to be 0.988916

The performance measures are

(i) Mean queue length $E(L)$ =0.172750
(ii) $E(L^2)$ =0.233721
(iii) Variance of queue length $L = var(L)$ =0.203878
(iv) Probability that the server is ideal =0.832079
(v) Mean queue length when the server is an vacation period =0.007599
(vi) Mean queue length when the server is regular busy period =0.219899
(vii) Probability that the server is in working vacation period $= pr\{J = 0\}$ =0.007243
(viii) Probability that the server is in regular busy period= $pr\{J = 1\}$ =0.141743

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