A Study on Fuzzy Graphs: Inverse Clique Regular Domination Number

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Abstract **–**If V – Dcr(G) contains clique regular dominating set D{\cr(G), then D{\cr(G) is called the inverse clique regular dominating set with regard to Dcr(G). A subset Dcr(G)of a fuzzy graph G=(σ,μ) is said to be a clique regular dominating set.The lowest fuzzy cardinality calculated over all minimal inverse clique regular dominating sets of G is known as the inverse clique domination number, or \cr(G)

Keywords –Fuzzy graphs, Fuzzy domination, , Clique regular domination, Inverse clique regular domination.

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1. Introduction

The concepts of regular domination and lique domination in graphs were first introduced by Kulli V.R. et al. [3]. The concept of a fuzzy graph was first proposed by Rosenfield, who also established a number of fuzzy analogues of graph theoretic ideas as path, cycles, and connectedness[9].The topic of dominance in fuzzy graphs was covered by A. and S. Somasundram [10]. The inverse clique regular dominance number in fuzzy graphs is discussed in this study, and the relationship with other well-known parameters of G.

2. Preliminaries

Definition:2.1

Let G=(V,E) be a graph. A subset D of V is called a dominating set in G if every vertex in V-D is adjacent to some vertex in D. The domination number of G is the minimum cardinatliy taken over all dominating sets in G and is denoted by γ(G).

Definition: 2.2

Let G=(σ, μ) be a fuzzy graph on V and $V_1 \subseteq V$. Define σ_1 on V_1 by $\sigma_1(u)=\sigma(u)$for all $u\in V_1$ and μ_1 on the collection E_1 of two element subsets of V_1 by $\mu_1(\{u ,v\}) = \mu(\{u ,v\})$ for all $u,v\in V_1$, then (σ_1,μ_1) is called the fuzzy subgraph of G induced by V_1 and is denoted by $<V_1>$.

Definition:.2.3

The fuzzy subgraph $H=(\sigma_1, \mu_1)$ is said to be a spanning fuzzy subgraph of G=(σ, μ) if $\sigma_1(u)=\sigma(u)$ for all $u\in V_1$ and $\mu_1(u, v)\le\mu(u, v)$ for all u,v∈V. Let G (σ, μ) be a fuzzy graph and σ_1 be any fuzzy subset of V_1, i.e. ,$\sigma_1(u)\le\sigma (u)$ for all u.

Definition: 2.4

Let G=(σ,μ) be a fuzzy graph on V. Let u, v∈V. We say that u dominates v in G if $\mu(\{u,v\})=\sigma(u)\wedge\sigma(v)$. A subset D of V is called a dominating set in G if for every v∉D, there exists u∈D such that u dominates v. The minimum fuzzy cardinality of a dominating set in G is called the domination number of G and is denoted by γ(G)orγ.

Definition: 2.5

A dominating set D of a fuzzy graph G is said to be a minimal dominating if no proper subset D′ of D is dominating set of G such that

$$\sum_{v_i\in D'} \sigma(v_i) < \sum_{v_i\in D} \sigma(v_i)$$

Definition: 2.6

The order p and size q of a fuzzy graph G=(σ,μ) are defined to be $p = \sum_{u\in V} \sigma(u)$ and $q = \sum_{(u,v)\in E} \mu(\{u ,v\})$.

Definition: 2.7

An edge e={u, v} of a fuzzy graph is called an effective edge if $\mu(\{u,v\}) = \sigma(u) \wedge \sigma(v)$.

$N(u) = \{ v\in V/ \mu(\{u ,v\}) = \sigma(u) \wedge \sigma(v)\}$ is called the neighborhood of u and $N[u]=N(u) \cup \{u\}$ is the closed neighborhood of u.

The effective degree of a vertex u is defined to be the sum of the weights of the effective edges incident at u and is denoted by dE(u). $\sum_{v \in N(u)} \sigma(v)$ is called the neighborhood degree of u and is denoted by dN(u). The minimum effective degree $\delta_E(G)=\min\{dE(u) / u \in V(G)\}$ and the maximum effective degree $\Delta_E(G) = \max\{dE(u) / u \in V(G)\}$.

Definition: 2.8

A vertex u of a fuzzy graph is said to be an isolated vertex if $\mu(\{u,v\}) < \sigma(u) \wedge \sigma(v)\}$ for all $v \in V-\{u\}$, that is , $N(u) = \phi$, Thus an isolated vertex does not dominate any other vertex in G.

Definition: 2.9

A set D of vertices of a fuzzy graph is said to be independent if $\mu(\{u,v\}) < \sigma(u) \wedge \sigma(v)\}$ for all u , v∈D.

Definition: 2.10

The complement of a fuzzy graph G, denoted by $\bar{G}$ is defined to be $\bar{G} = (\sigma, \bar{\mu})$ where $\bar{\mu}(\{u,v\}) = \sigma(u) \wedge \sigma(v) - \mu(\{u,v\})$.

Definition: 2.11

Let σ: V→ [0, 1] be a fuzzy subset of V. Then the complete fuzzy graph on σ is defined to be (σ,μ) where $\mu(\{u,v\}) = \sigma(u) \wedge \sigma(v)$ for all uv∈E and is denoted by K_σ.

Definition: 2.12

A fuzzy graph G=(σ,μ) is said to be bipartite if the vertex V can be partitioned into two nonempty sets V_1 and V_2 such that $\mu(v_1,v_2)=0$ if $v_1,v_2 \in V_1$ or $v_1,v_2 \in V_2$. Further, if $\mu(u,v)=\sigma(u) \wedge \sigma(v)$ for all $u \in V_1$ and $v \in V_2$ then G is called a complete bipartite graph and is denoted by K_{σ_1,σ_2} where σ_1 and σ_2 are the restrictions of σ to V_1 and V_2 respectively.

Definition: 2.13

Let G = (σ, μ) be a regular fuzzy graph on $G^* = (V, E)$. If $d_G(v) = k$ for all v∈V, (i.e ,) if each vertex has same degree k, then G is said to be a regular fuzzy graph of degree k or k-regular fuzzy graph. Where $G^* = (V, E)$ is an underlying crisp graph.

Remark: 2.14

G is k-regular graph iff δ=Δ=k.

Definition: 2.15

Let G= (σ, μ) be a fuzzy graph. The total degree of a vertex u∈V is defined by $td_G(u) = d_G(u) + \sigma(u) = \sum_{uv \in E} \mu(uv) + \sigma(u)$.

If each vertex of G has the same total degree k then G is said to be a totally regular fuzzy graph of total degree k or k-totally regular fuzzy graph

Definition: 2.16

A set of fuzzy vertex which covers all the fuzzy edges is called a fuzzy vertex cover of G and the minimum cardinality of a fuzzy vertex cover is called a vertex covering number of G and denoted by β(G).

Definition: 2.17

Let G = (σ, μ) be a fuzzy graph on D and D ⊆ E then the fuzzy edge cardinality of D is defined to be $\sum_{e \in D} \mu(e)$.

Definition: 2.18

The effective degree of a vertex u is defined to be the sum of the weights of the effective edges incident of 'u' and is denoted by dE(u). $\sum_{v \in N(v)} \sigma(v)$ is called the neighbourhood of u and is denoted by dN(u).

Definition: 2.19

The minimum effective degree $\delta_E(G) = \min\{dE(u) \mid u \in V(G)\}$ and the maximum effective degree $\Delta_E(G) = \max\{dE(u) \mid u \in V(G)\}$.

3. MAIN RESULTS

Definition 3.1

Let G = (σ, μ) be a fuzzy graph without isolated vertices. A subset $D_{cr}(G)$ of V is said to be a clique regular dominating set if V – $D_{cr}(G)$ contains clique regular dominating set $D'_{cr}(G)$ then $D'_{cr}(G)$ is called the inverse clique regular dominating set with respect to $D_{cr}(G)$ .The inverse clique domination number $\gamma'_{cr}(G)$ is the minimum fuzzy cardinality taken over all minimal inverse clique regular dominating sets of G.

Example 3.2

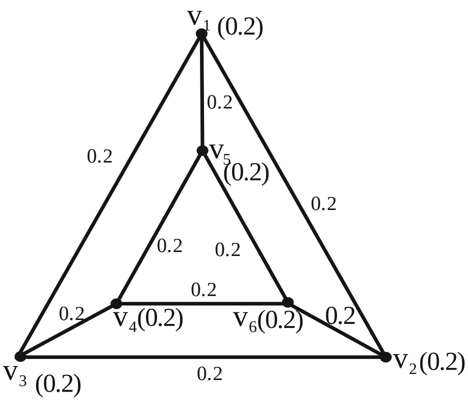

Fig.1

$D_{cr}(G) = \{v_1, v_2, v_3\}$ $\gamma_{cr}(G) = 0.6$

$D'_{cr}(G) = \{v_4, v_5, v_6\}$ $\gamma'_{cr}(G) = 0.6$

Theorem 3.3

If $G = (\sigma, \mu)$ is a complete fuzzy graph K_σ with $n \geq 2$ then

(i) $<N(D_{cr}(G))>$ is a fuzzy complete graph with (n–1) vertices.

(ii) $< N (D'_{cr}(G)) >$ is a fuzzy complete graph with (n–2) vertices

Proof:

Let $G = (\sigma, \mu)$ be a complete fuzzy graph K_σ with $\sigma(v_i) = c$, for every $v_i \in V$ and $n \geq 2$. $D_{cr}(G)$ is the fuzzy clique regular dominating set. Clearly $D_{cr}(G) = \{ v_i \}$ and $< N(D_{rc}(G)) >$ is a complete fuzzy graph with (n-1) fuzzy vertices. Clearly $< N(D_{rc}(G)) >$ is a complete fuzzy graph with (n–1) fuzzy vertices., further $V- D_{cr}(G) = V- \{ v_i \} = \{ v_1,v_2,\ldots,v_{i-1},v_{i+1},\ldots,v_n\}$.

Let $D'_{cr}(G) \subseteq V- D_{cr}(G)$ is the fuzzy inverse clique regular dominating set then $D'_{cr}(G) = \{ v_j / \sigma(v_i)$ is minimum, $j \neq i\}$,also $< V- D'_{cr}(G) >$ is regular with vertices of degree (n–2)c. Moreover, $<N(D'_{rc}(G))>$ is complete with (n–2) fuzzy vertices. Therefore, $< N (D'_{cr}(G)) >$ is a complete fuzzy graph with (n–2) intuitionistic fuzzy vertices.

Theorem 3.4

If $G = (\sigma, \mu)$ is a fuzzy cycle with equal fuzzy vertex cardinality and $\gamma'_{cr}(G)$ - set exist, then $< N(D_{cr}(G)) >$ is a fuzzy complete graph with two vertices.

Proof:

$G = (\sigma, \mu)$ be a fuzzy cycle with vertex set $V = \{ v_1,v_2,\ldots,v_i,v_{i+1},\ldots,v_n= v_0\}$ such that v_i is adjacent with $v_{(i-1) \bmod n}$ and $v_{(i+1) \bmod n}$ $1 \leq i \leq n$. Moreover, v_i dominates $v_{(i-1) \bmod n}$ and $v_{(i+1) \bmod n}$. Let $D_{cr}(G)$ be the clique regular dominating set with (n-2) vertices such that $< N(D_{rc}(G)) >$ is regular and also complete graph with two fuzzy vertices. Therefore, $<N(D_{rc}(G))>$ is a fuzzy complete graph with two vertices.

Theorem 3.5

If $G = (\sigma, \mu)$ is a fuzzy cycle and $\sigma(v_i)$'s are constant with $\mu (v_i, v_j) = \min \{ \sigma(v_i), \sigma(v_j) \}$ then $\gamma'_{cr}(G) = (n–2) \sigma(v_i)$.

Proof:

Let $G = (\sigma, \mu)$ be a fuzzy cycle with vertex set $V = \{ v_1,v_2,\ldots,v_i,v_{i+1},\ldots,v_n= v_0\}$ such that v_i is adjacent with $v_{(i-1) \bmod n}$ and $v_{(i+1) \bmod n}$ $1 \leq i \leq n$. Moreover, v_i dominates $v_{(i-1) \bmod n}$ and $v_{(i+1) \bmod n}$ and $\sigma(v_i) = c$ for every $v_i \in G$ with $\mu (v_i, v_j) = \min \{ \sigma(v_i), \sigma(v_j) \}$, then by theorem 2.7.4 $< N(D_{cr}) >$ is a fuzzy complete graph with two vertices , clearly $D'_{cr}(G)$ has (n–2) fuzzy vertices. Therefore, the fuzzy clique regular domination number $\gamma'_{cr}(G) = (n–2) \sigma(v_i)$.

Theorem 3.6

If $G = (\sigma, \mu)$ is a fuzzy cycle with all effective edges, then $\gamma'_{cr}(G) = p – \max \{ \sigma(v_i) + \sigma(v_{i+1})\}$.

Proof:

$G = (\sigma, \mu)$ be a fuzzy cycle with vertex set $V = \{ v_1,v_2,\ldots,v_i,v_{i+1},\ldots,v_n= v_0\}$ such that v_i is adjacent with $v_{(i-1) \bmod n}$ and $v_{(i+1) \bmod n}$ $1 \leq i \leq n$. Moreover, v_i dominates $v_{(i-1) \bmod n}$ and $v_{(i+1) \bmod n}$. G is a fuzzy cycle of all effective edges then by theorem 2.7.4, $D_{cr}(G)$ has (n-2) fuzzy vertices. $V – D_{cr}(G)$ has a complete fuzzy graph with two fuzzy vertices. The fuzzy clique regular dominating set is the minimum fuzzy cardinality taken over all fuzzy clique regular dominating sets of G.

Therefore, $\gamma'_{cr}(G) = p – \max \{ \sigma(v_i) + \sigma(v_{i+1})\}$.

Theorem 3.7

If $G = (\sigma, \mu)$ is a fuzzy path with all effective edges then $\gamma'_{cr}(G) = p – \max \{ \sigma(v_i) + \sigma(v_{i+1}) / i \neq 1 \text{ or } n\}$

Proof:

Let $G = (\sigma, \mu)$ be a fuzzy path with vertex set $V = \{ v_1,v_2,\ldots,v_i,v_{i+1},\ldots,v_n\}$ and having all effective edges, v_i's are adjacent with v_{i+1} also v_i dominates v_{i+1}, i= 1 to n–1. Let $D_{cr}(G)$ be the clique regular dominating set which contains $\{ v_i / v_i \in G\}$ such that $< N(D_{cr}(G)) >$ is regular. The minimum fuzzy clique regular domination number $\gamma'_{cr}(G) = p – \max \{ \sigma(v_i) + \sigma(v_{i+1}) / i \neq 1 \text{ or } n\}$.

Theorem 3.8

If $G = (\sigma, \mu)$ is a fuzzy wheel W_{n+1} with $\sigma(v_i) = c$, for every $v_i \in V$ and all edges are effective then $\gamma'_{cr}(G) = \{\sigma(v) / v$ is the centre vertex of the fuzzy wheel$\}$.

Proof:

Let $G = (\sigma, \mu)$ be a fuzzy wheel W_{n+1} with $\sigma(v_i) = c$, for every $v_i \in V$ and having all effective edges. The vertex set of G is $\{ v, v_1, v_2, \ldots, v_i, v_{i+1}, \ldots, v_n \}$ where v is the centre vertex of the fuzzy wheel, v is adjacent with v_i, i= 1 to n also v dominates v_i i= 1 to n. Further, v_i is adjacent with $v_{(i-1) \bmod n}$ and $v_{(i+1) \bmod n}$ $1 \le i \le n$ and v_i dominates $v_{(i-1 \bmod n)}$ and $v_{(i+1 \bmod n)}$ $1 \le i \le n$. Let $D_{cr}(G)$ be the clique regular dominating set which contains { v / v is the centre vertex of the fuzzy wheel} such that $< N(D_{cr}) >$ is regular. Therefore, The minimum fuzzy clique regular domination number $\gamma_{cr}(G) = \sigma(v) = c$, v is the centre vertex of the fuzzy wheel.

Theorem 3.9

If $G = (\sigma, \mu)$ is a fuzzy wheel W_{n+1} with all edges are effective then $\gamma'_{cr}(G) < p - k$ where $k = \sigma(v_i) + \sigma(v_{i+1})$ is maximum.

Proof:

Let $G = (\sigma,\mu)$ be a fuzzy wheel W_{n+1} with vertex set $\{v, v_1, v_2 \ldots, v_n = v_0\}$ such that v is adjacent with $v_1, v_2, \ldots, v_n = v_0$ and having all effective edges. Let $D_{cr}(G)$ = {v/v is the centre of the vertex} is the clique regular dominating set since $<N(D_{cr}(G)>$ is regular and $D'_{cr}(G) = V - \{v_i, v_{i+1}\}$ such that $< D'_{cr}(G)>$ is regular. The inverse clique regular dominating number $\gamma'_{cr}(G)$ is $p - \max\{\sigma(v_i) + \sigma(v_{i+1})\}$ such that $\gamma'_{cr}(G)$ is the minimum inverse clique regular dominating set of G.

Theorem 3.10

If $G = (\sigma, \mu)$ is a fuzzy wheel W_{n+1} with $\mu(v_i, v_j) = \min\{\sigma(v_i), \sigma(v_j)\}$ and $\sigma(v_i) = c$, for every $v_i \in V$ then $\gamma'_{cr}(G) = p - 2c$.

Proof:

Let $G = (\sigma,\mu)$ be a fuzzy wheel W_{n+1} with vertex set $\{v, v_1, v_2, \ldots v_n = v_0\}$ and having $\mu(v_i,v_j) = \sigma(v_i) \wedge \sigma(v_j)$. Let $D_{cr}(G)$ = {v/v is the centre of the wheel} and $D'_{cr}(G) = V - \{v_i, v_{i+1}\}$ such that $<N(D'_{cr}(G))>$ is regular, which is K_2. Therefore, inverse clique regular domination number $\gamma'_{cr}(G)$ then by theorem 2.7.9, $\gamma'_{cr}(G) = p - \max\{\sigma(v_i) + \sigma(v_{i+1})\}$ since $\sigma(v_i) = c$, for every $v_i \in V$ clearly $\gamma'_{cr}(G) = p - 2c$.

Corollary: 3.11

If all edges are not effective in a fuzzy path $G = (\sigma, \mu)$ then $\gamma'_{cr}(G)$ does not exist.

Theorem 3.12

If $G = (\sigma, \mu)$ is a complete fuzzy graph then $\gamma_{cr}(G)$ - set and $\gamma'_{cr}(G)$ - set exists but converse need not be true.

Proof:

If $G = (\sigma, \mu)$ is a complete fuzzy graph K_σ with vertex set $\{v_1, v_2, \ldots, v_i, v_{i+1}, \ldots, v_n\}$ and every $v \in V$ is adjacent to the other vertices. Further, every v dominates the other vertices. Suppose the degree of all $v_i \in V$ are equal. Then the fuzzy clique regular dominating set exists, therefore $\gamma_{cr}(G)$ - set exists.

Converse need not be true since $\gamma_{cr}(G)$ - set exists, the degree of $v_i \in V$ are equal, but all v_i's need not adjacent with other vertices, further all v_i's need not dominate the other vertices. By definition of complete graphs, G is not a fuzzy complete graph.

Example 3.13

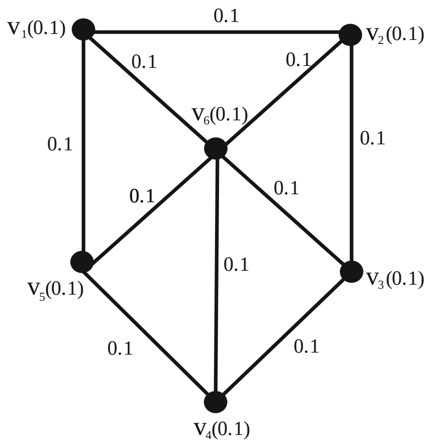

Fig.2

$D_{cr}(G) = \{v\}$, $\gamma_{cr}(G) = 0.1$

$D'_{cr}(G) = \{v_3, v_4, v_5\}$, $\gamma'_{cr}(G) = 0.3$

$\therefore \gamma_{cr}(G)$ and $\gamma'_{cr}(G)$ exist but G is not a complete graph.

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